# MATH 448/COMP 448

# CONCRETE MATHEMATICS

Fall 2018 MWF 2-2:50, HBH 227

**Instructors:** Ron Goldman, Professor of COMP and Frank Jones, Professor of MATH

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**Text: Graham, Knuth, & Patashnik, *Concrete Mathematics, 2nd ed*, Addison-Wesley**
Available in the book store, also available as a PDF.

**Goal:** To learn as much as possible from this excellent book, and at the same time to develop a definite *joy* in accomplishing prodigious mathematical feats. This course is definitely not concerned with memorizing formulas, but rather with acquiring exciting techniques for approaching and solving problems associated to recursion, summation, and elementary number theory. We shall skip some sections, but we definitely plan to cover completely Chapter 7, **Generating Functions**.

**Prerequisite:** MATH 102. This modest prerequisite comes with the observation that this course has a 400 number. This course will really stretch and strengthen your (*our*) calculus skills! Analogies between discrete mathematics and continuous mathematics (calculus) will be emphasized.

COMP 182 is also a prerequisite for COMP majors. COMP majors need to have at least a modest background in discrete mathematics before taking this course.

**Homework:** Homework will be due on Fridays. You are allowed to work together, but you should be sure to try everything on your own as much as possible. All exercises in the book have solutions at the back of the book. You should try each exercise on your own before reading the book's solution. You may also look for solutions on the internet. The HW you hand in should be written out by you individually and should not merely be copied from a solution you have found. If you use the internet, you should cite the location you used.

**One Midterm Exam:** This exam will be open book with no time limit.

**Final Exam:** This exam will be open book and will have a time limit. It will be due Wednesday December 12.

**Exams Rules:** Unlike the homework, on the midterm exam and on the final exam you are not allowed to use the internet or any sources other than the textbook, and you are certainly not allowed to collaborate; *everything you submit must be entirely your own work.*

**Exercises:** There are over 500 exercises in the book. As the back cover states, "Complete answers are provided for all exercises, except research problems, making the book particularly valuable for self-study."

That last sentence presents quite a challenge for us! We have to find a way to assess your progress and prowess, but all the answers to all the exercises are in the book. And this is not supposed to be a "self-study" course. We think we'll be able to come to some clear conclusions about your individual understanding.

**Grading:** HW 50%, Midterm Exam 25%, Final Exam 25%

Meanwhile, here are three exercises that may help get you in the right frame of mind for the semester:

**A.** Define $x_0 = 1$ and for $n \geq 0$ let $x_{n+1} = 3x_n + \lfloor x_n\sqrt{5} \rfloor$. Find an explicit formula for $x_n$ for all *n*.
(Putnam Competition 2007)

This exercise is due Friday September 28.

**B.** Look at the Fibonacci sequence as defined on page 291. Compute the infinite product of the quotients $(F_{2n} + 1)/(F_{2n} - 1)$ for $n$=2,3,4, … . (You will probably want to approach this as follows:

a. Compute a few finite products, and guess a pattern that holds for every finite product of terms for $n = 2, 3, 4, \ldots, N$.
b. Prove that your pattern is actually valid for all N.
c. Finally, let $N \to \infty$

(Frank saw this problem in *Mathematics Magazine*, vol. 79, No. 5, December 2006, p. 393.)

This exercise is due Friday October 26.

**C.** Frank's daughter has a puzzle that consists of a board with 7 holes in a line, 3 white pegs, and 3 black pegs. At the start, the pegs are arranged in the line WWW – BBB. One is supposed to move the pegs to the state BBB – WWW, using two types of moves: slide a peg from one hole to an adjacent hole, or jump over one peg to land in the empty hole. (For instance, the first move might result in WW – WBBB or it might result in W – WWBBB.) This is easy enough to accomplish, and I can solve this puzzle in 15 moves. The question for you is this: for any positive integer *n*, suppose you have a puzzle with *n* white pegs, *n* black pegs, and a board with $2n+1$ holes in a line. Determine, *with proof*, the smallest number of moves required to switch the white and black pegs.

This exercise is like a research problem. We hope some of you can do it.